A New Solution for 7x7 Hex Game

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Abstract. In this paper, we applied the decomposition method to obtain a new winning strategy for 7x7 Hex game. We also find that some positions on the 7x7 Hex board, which are called "trivial positions", were never occupied by Black among all of strategies in the new solution. In other words, Black can still win the game by using the strategies described in this paper even if White already has pieces on those positions. Considering the symmetry properties of a Hex board for both players, we derived 14 losing positions on a 7x7 Hex board for Black's first move.

1 Introduction

Hex is an interesting board game, which was invented by Piet Hein, a Danish mathematician, in 1942. It was reinvented independently by mathematician John Nash in 1948. The game is played by two players taking turns to play their pieces on the unoccupied cells of the board. The object of the game is to build a connected chain of pieces across opposite sides of the board. The Hex board is a hexagonal tiling of n rows and m column, while n is usually equal to m. Figure 1 is an empty Hex board with size 7x7.

The rules of the game are relatively simple:

- Players take turns playing a piece of their color on an unoccupied hexagon.
- Player with black pieces plays first.
- The game is won when one player establishes an unbroken chain of their pieces connecting their sides of the board.

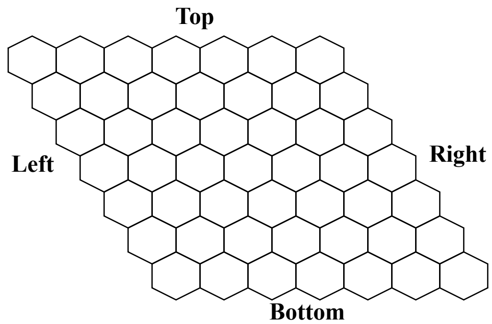

Fig. 1. An empty 7x7 Hex board

For example, Figure 2 is in the middle stage of a Hex game and it is Black's turn to play. If Black plays a piece at position "A", Black wins the game. However, if Black plays at any other position rather than "A", White will play at position "A" and declare the win.

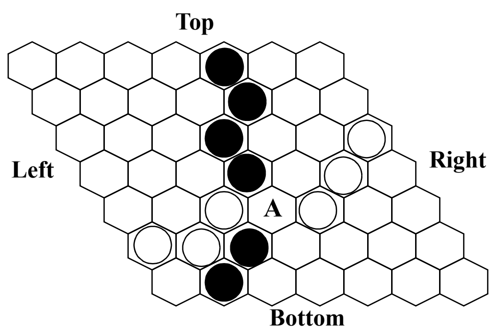

Fig. 2. Play on position "A" to win the game.

In 1949, John Nash proved that there is no tie in a Hex game and the player who plays first has a theoretical win, though the proof does not indicate a winning strategy. A winning strategy based on the decomposition method for the Hex game played on a 7x7 board was declared in 2001 [1]. In the solution, the first move is on the center of the Hex board in order to take the advantage of the symmetry properties.

In this paper, a new solution based on the decomposition method for Hex 7x7 is described. This research also leads to the finding of some "losing positions" for Black's first move, which will lead Black to a lose if White adopts the existing winning strategy for Black.

2 A New Solution for Hex on 7x7 Board

In the first winning strategy developed for the Hex game played on a 7x7 board [1], the first piece is played at the center of the Hex 7x7 board to take the advantage of the symmetry properties. In this paper, we describe a new solution for the 7x7 Hex game, in which the first piece is played at the position ➊ as shown in Figure 3.

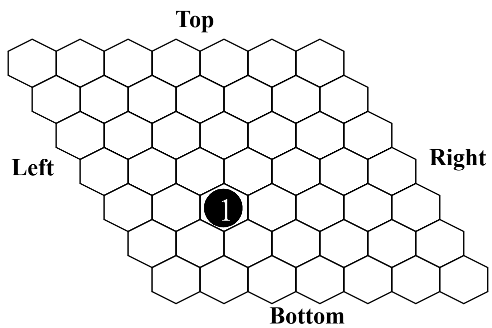

Fig. 3. The first move in the new solution. This is also the LocalPattern 1 in [2].

The decomposition method is inspired from the subgoal idea in AI planning. In a Hex game, the goal for Black is to form a connected chain from the top side to the bottom side on the board. Sometimes, the goal can be viewed as a conjunctive goal of several subgoals. A subgoal may be "one Black piece connected with another Black piece", "one piece connected with the top row", "one Black piece connected with the bottom row", "the top row connected with the bottom row" or a combination of their OR/AND logical expressions. For example, "one Black piece connected with another Black piece" OR "this Black piece connected with the top row" is one of the typical cases. The successes of achieving all of the subgoals will lead to the success of accomplishing the goal. An important characteristic of the Hex game is that the success of a subgoal may only be determined by a small empty local region, which is called the influence region of the subgoal. If each of those subgoals has an influence region for the subgoal's success and all influence regions in a game are independent from each other without any overlap, we can decompose the entire board into several local patterns. For example, in Figure 4, the game can be decomposed into three different local regions, each of them has a subgoal. The subgoal of local pattern ✧ is to connect ➌ with the top row, its influence region covers all ✧s; The subgoal of Local Pattern X is to connect ➊ with ➌, its influence region is over all Xs; The subgoal of Local Pattern △ is to connect ➌ with the bottom row, its influence region is marked by △s. Obviously, the three influence regions do not overlap each other, and Black can win the game by forming a connected chain from the top row to the bottom row if all of three subgoals are reached. If we find the strategies on each local regions for Black to reach its subgoal, putting those strategies together will be the strategies for Black to win the game. Since White's move can be in only one of the local pattern regions, Black only needs to play the next move using the strategies for the corresponding local pattern.

In this paper, we only view the new solution from the top two levels, though all detailed playing strategies for the new solution, including 63 local patterns, can be referenced from [2] at `http://www.ee.umanitoba.ca/~jingyang/TR.pdf`.

In each of the following figures from Figure 4 to Figure 19, different local influence regions are labeled by different marks, for example, X, △ or ✦ and so on. Each local patterns also has a local pattern number (LocalPattern n) that is used in [2].

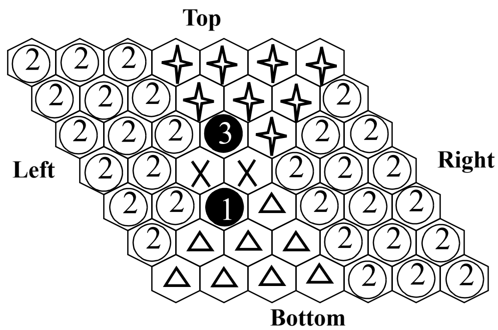

Fig. 4. ✦s are the LocalPattern 5 and their subgoal is to connect ❸ to Top; Xs are the LocalPattern 2 and their subgoal is to connect ❸ with ❶; and △s are the LocalPattern 5 and their subgoal is to connect ❶ to Bottom. Black will win by playing ❸ if white plays any one of ② positions. We will use this kind of representations through this paper.

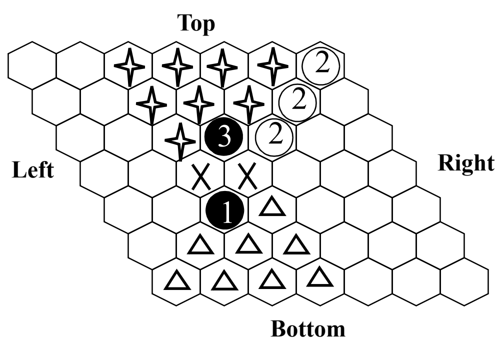

Fig. 5. ✦s, Xs, and △s indicate the LocalPattern 5, the LocalPattern 2, and the LocalPattern 5, respectively.

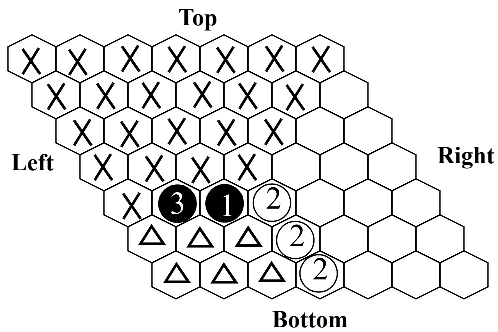

Fig. 6. Xs form the LocalPattern 20, and △s are the LocalPattern 6.

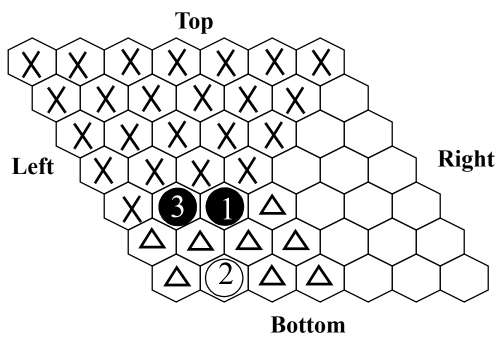

Fig. 7. Xs and △s indicate the LocalPattern 20 and the LocalPattern 4, respectively.

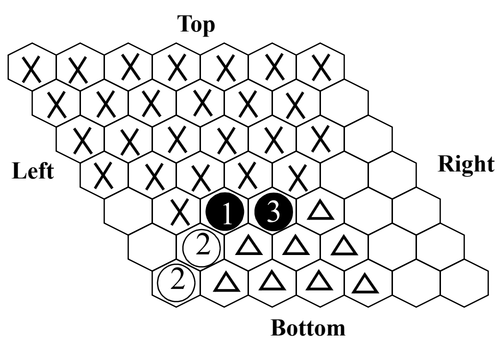

Fig. 8. Xs and △s indicate the LocalPattern 19 and the LocalPattern 5.

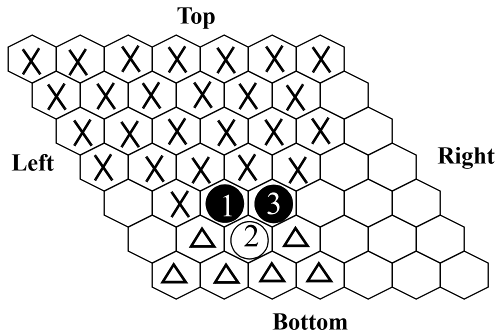

Fig. 9. X s and △ s indicate the LocalPattern 19 and the LocalPattern 3, respectively.

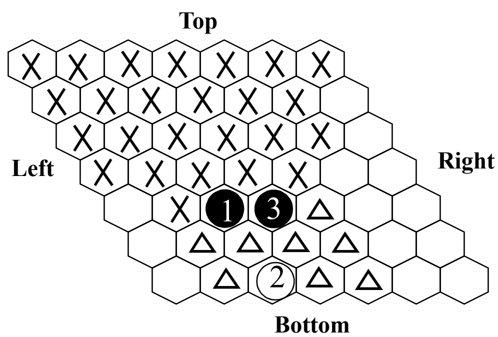

Fig. 10. X s form the LocalPattern 19 and △ s indicate the LocalPattern 4.

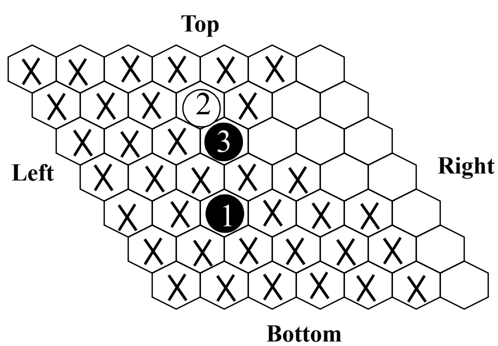

Fig. 11. X s indicate the LocalPattern 11.

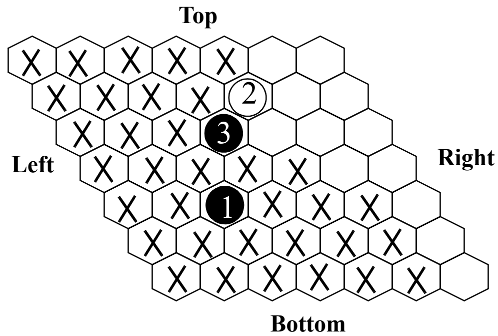

Fig. 12. X s form the LocalPattern 12.

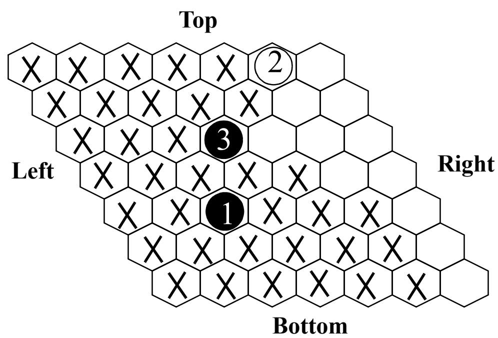

Fig. 13. X s form the LocalPattern 14.

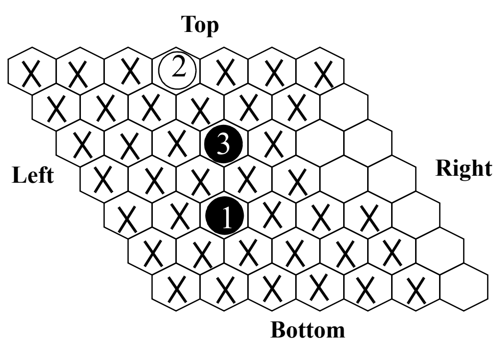

Fig. 14. X s indicate the LocalPattern 15.

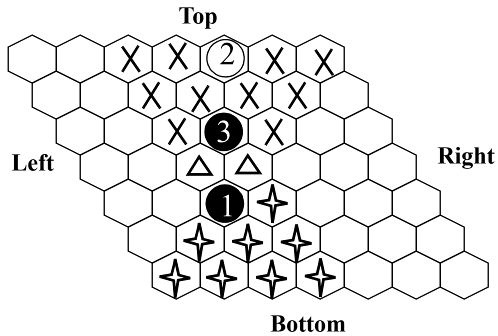

Fig. 15. Xs, △s, and ✧s form the LocalPattern 9, the LocalPattern 2, and the LocalPattern 5, respectively.

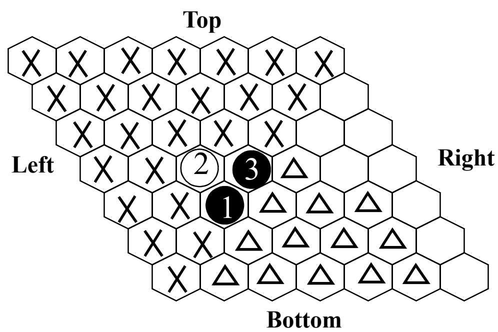

Fig. 16. Xs form the LocalPattern 41 and △s indicate the LocalPattern21.

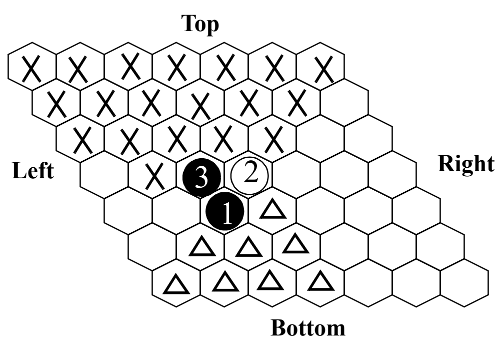

Fig. 17. Xs indicate the LocalPattern 34 and △s form the LocalPattern5.

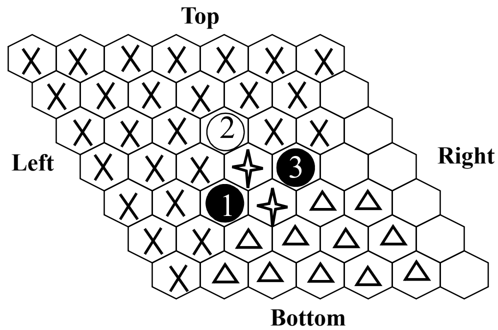

Fig. 18. Xs are the LocalPattern 22; △s are the LocalPattern21; and ✦s are the LocalPattern 2. White's ② is the strongest defend move in the new solution. But Black's ❸ divides the board into three local patterns and win the game. The Local pattern ✦ makes sure of Black's ❶ piece connected with Black's ❸ piece; The Local pattern △, guarantees Black's ❶ and ❸ piece connecting with Bottom; and The Local Pattern X is responsible for connecting Top with Black's ❶ and ❸ piece or directly with Bottom.

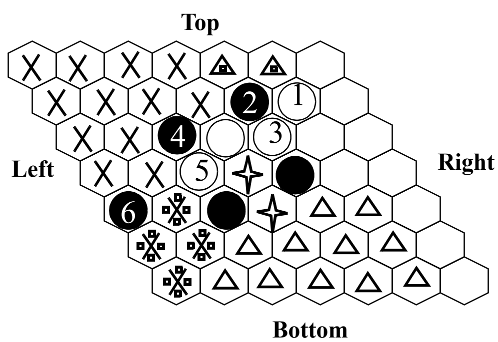

Fig. 19. Further developed from Figure 18. Xs, ⟁, ✦, ❉ and △ are the LocalPattern 25, the LocalPattern 2, the LocalPattern 2, the LocalPattern 13, and the LocalPattern 21 respectively.

3 Losing Opening Moves for Black

Although there are winning strategies for the first player to win a Hex game, a "bad" opening move can lead to a lose. In a solution for the Hex game, if there are some positions that Black never needs to occupy, we define those positions as "trivial positions". In other words, even if White has some pieces on those "trivial positions", Black can still play with the strategies in the solution without being affected. For example, in Figure 20, all positions marked by X are "trivial positions", which were never needed by Black in the solution aforementioned. Black can win the game by following the winning strategies in the solution regardlessly even if White has played pieces on those positions.

If the Hex board is turned over along the diagonal (from left-top corner to right-bottom corner), we would find that all of these marked positions in Figure 20 then become Black's losing positions.These positions are marked by ❶ and shown in Figure 21. If Black occupies one of those positions with the first piece, White can play at ② and follow the same winning strategy proposed in this paper and win the game.

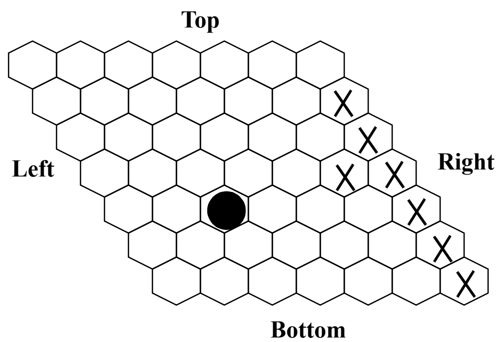

Fig. 20. All of X s are trivial positions. Even if they are all occupied by White's pieces, Black still will form a connected chain from Top to Bottom and win the game.

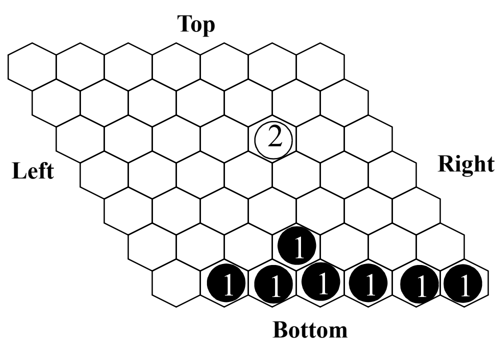

Fig. 21. ❶s are losing positions for Black. If Black plays its first move on any one of ❶s, White will play ② and win the game. Note that, White will play exactly what Black does in the new solution in the section 2 except White's goal is to form a connected chain from Left to Right. If you draw a diagonal line from left-top corner to right-bottom corner, and then use the diagonal line as the axis to turn over the board, the situation will be the same as the Figure 20.

4 Conclusions

In this paper, we described a new winning strategy for the Hex game played on a 7x7 board. Due to the subgoals of winning a game in which the first piece is played at ❶ in Figure 22, we decompose the game into 63 local patterns. Each local pattern will ensure a subgoal being achieved.

Through Figure 4 to Figure 18 (position A to O in Figure 22), all possible moves by White are considered on the second move level, while Black is going to win the game regardlessly. We conclude that Black ❶ in Figure 22 is a winning move.

Due to the symmetry properties of the 7x7 Hex board, we also conclude that ❶ in Figure 23 is also a winning move. The winning strategy described in this paper can ensure the win with some simple coordinate transformations.

According to the "trivial positions" discovered in the new winning strategy and the symmetry properties of a 7x7 Hex board, we derived 14 losing opening positions for Black, which are shown in Figure 24.

The newly discovered losing positions are especially valuable when the "Swap Rule", which gives the player to move second the option of swapping colors after Black's first move, is applied in a Hex game.

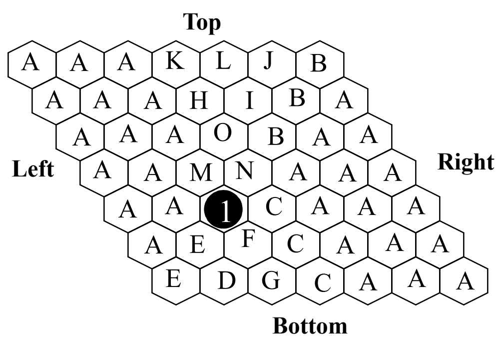

Fig. 22. Position As correspond to Figure 4, position Bs correspond to Figure 5, position Cs correspond to Figure 6,......, position N corresponds to Figure 17 and position O corresponds to Figure 18. It is obvious that all White's defend moves are covered in the new solution.

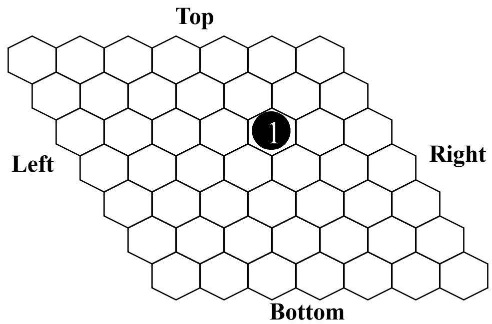

Fig. 23. Due to the symmetry properties of the 7x7 Hex board, ❶ is also a winning move.

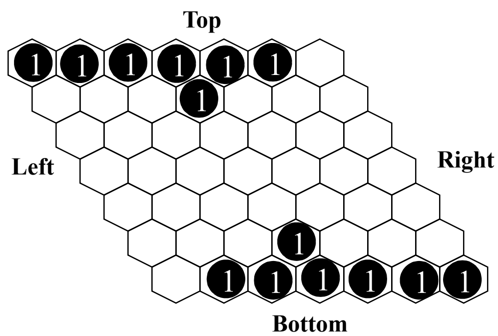

Fig. 24. Losing positions for Black derived from the new solution.

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